MATH 105 101 Midterm 2 Sample 5

1. (20 marks)

   (a) (5 marks) Determine the intervals on which the following function is concave up or concave down:

   $$F(x) = \int_0^x te^t \, dt.$$

   (b) (5 marks) Find the definite integral:

   $$\int_{-\pi/2}^{\pi/2} \sin^7(x) \, dx.$$

   (c) (5 marks) Compute the Right Riemann sum for $f(x) = \sin^2(x)$ on the interval $[0, \pi]$ using $n = 6$ equal subintervals. **Simplify the answer.**

   (d) (5 marks) Find the definite integral

   $$\int_{-\infty}^0 e^{3x} \, dx.$$

2. (10 marks) Let $f(x) = x^3 - \cos(x)$ on $[-\pi, \pi]$.

   (a) (5 marks) Compute an approximation of $\int_{-\pi}^{\pi} f(x) \, dx$ using Simpson's Rule with $n = 4$ equal subintervals. **Simplify the answer.**

   (b) (5 marks) Find an upper bound of the error for the approximation in part (a).

3. (10 marks) Evaluate the indefinite integral:

   $$\int \cos(\ln x) \, dx.$$

4. (10 marks) Solve the initial value problem:

   $$\frac{dy}{dt} = \frac{ty^3}{\sqrt{1 + t^2}}, \qquad y(0) = -1.$$

   Express the answer in its _explicit_ form.